**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Ans: B.0.2676

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans: A. False

B. True

A. Mean=38

SD=6

Z score= (value-Mean)/SD

Z score for 44= (44-38) =1=84.13%

Above 44 age =100-84.13=15.87%=63 out of 400

Z score for 38 = (38-38)/6=0=50%

Hence people between 38 and 44 age=84.13-50=34.13%=137 out of 400, so the statement is false

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Ans: The difference between 2X1 and X1+X2 is N (0, 6 σ2)

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Ans: Z value at 0.5th percentile is given as=-2.576

Z value at 99.5 percentile is given as=2.576

Z=(X-100)/20=>x=20z+100

a=-(20\*2.576)+100=48.5

b=(20\*2.576)+100=151.5

Two values symmetric about mean for the given standard normal distribution are[48.5,151.5]

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

Sol:

**a.95% of the probability lies between 1.96 standard deviation of the mean**

**Range is:**

= variance of the company distribution= σ^2=32+42=9+16=25=52

Standard devation of the company distribution= σ=√(5^2)=5

Confidence level=CL=0.95

Confidence level=Cl=μ±Za/2=0.025(σ)

=540±1.96(225)

=99,981 in millions

**b. Fifth percentile is calculated as:**

5th percentile of profit is Rs.170.1 million

**c. the division that has a larger probability which division is giving less value that loss is an higher loss**

The Probability of Division #1 making a loss is 4.78 %

The Probability of Division #2 making a loss is 4.01 %

The Division 1 has a larger Probability of making a loss